Quantity Discounts from Risk Averse Sellers

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Federal Trade Commission*
January 26, 2009

*The opinions expressed are not necessarily those of the Commission nor any Commissioner

Introduction

- Can larger customers command lower prices than smaller customers?
  - No cost difference from serving customers of different sizes
  - No incentive for price discrimination
  - No monopsony power
- Explain GPO consolidation?

Literature

- Current explanations
  - Snyder
    - Supports tacit collusion among sellers as an equilibrium
  - Chae & Heidhues, Stole & Zweibel, Chipty & Snyder
    - Negotiator's split evenly the surplus of negotiation.
    - Benefits are concave in output.
    - Even division implies lower price for big customers.
  - Ellison and Snyder
    - Pharmaceutical monopolists do not lower price to large pharmacies

My Analysis - Intuition

- Seller
  - risk averse - maximizes expected utility.
  - can tell large customers from small customers.
  - cannot observe customer valuations.
- In this market large customers are riskier than small customers.
- The seller offers lower prices to larger customers to mitigate additional risk.

My Analysis - Effects

- Pure concentration effect
  - One customer buying 5 units is riskier than five customers buying 1 unit each.
  - Large customer gets lower price than 5 small ones.
- Market size effect
  - Constant relative risk aversion implies increasing all customers' sizes proportionately has no effect on price.
- Customer mix effect
  - The price a customer sees depends on the revenue received from other customers.
  - This could cause a large customer to receive a higher price than a smaller customer.

Benchmark Model

- Customers
  - Each customer demands a fixed quantity, m_i
  - Customers' per unit value i.i.d. uniform [0, 1]
- Seller
  - Observes each customer's quantity
  - Cannot observe customers' valuation
  - Risk neutral
  - Sets customer specific prices

Benchmark Model

- $\Pi = \sum_i (1-p_i) p_i m_i$
- Optimal price is ½ for all i
- Optimal price is independent of m_i

Extension of the Model

- Seller has concave utility function
- Maximizes expected utility
- Two questions
  - Compare same sized markets with different numbers of identical customers
  - Examine a single market with different sized customers.

Pure Concentration Effect

- Customers value the same number of units
- Overall market demand held constant
- Compare price in a market with large customers to price in one with small (more) customers.
  - Consistent with GPOs
- Price for larger customers lower than price for smaller customers.

Pure Concentration Effect

- ***Proposition 1***: One customer of mass 1 receives a lower price than a continuum of mass 1.
  - For a continuum: $E(U) = U((1-p)p)$
    - $p^* = ½$
  - For a single customer: $E(U) = (1-p)U(p)$
    - FOC $(1-p)U'(p) - U(p) = 0$
    - Evaluate at $p = ½$
    - $(½)U'(½) - U(½) < 0$ by concavity of U.

Pure Concentration Effect

- ***Proposition 1, Intuition***:
- With a continuum of customers the seller receives (1-p)p for certain so there is no risk. So she sets the optimal risk neutral price of ½.
- With one customer the seller either receives 0 (with probability p) or p (with probability 1-p) So the seller lowers the price relative to the risk neutral price to reduce risk. (Trades off some marginal revenue in the good state for an increased chance of the good state happening. The concave utility function lowers the value of marginal revenue in the good state relative to the risk netral decision maker.)

Pure Concentration Effect

- ***Proposition 2***: One customer of mass 1 receives a lower price than 2 customers of mass ½ each.
  - For a single customer: $E(U) = (1-p)U(p)$
    - p^* solves FOC $(1-p)U'(p) - U(p) = 0$
  - For 2 customers: $E(U) =$
  - $(1-p_1)(1-p_2)U(p_1/2+p_2/2) + (1-p_1)p_2U(p_1/2) + p_2(1-p_1)\, U(p_1/2)$
    - FOC simplifies to
  - $(1-p^*)[U(p^*/2)-U(p^*)(½)]+p^*[(1-p^*)U'(p^*/2)(½)-U(p^*/2)] > 0$

Pure Concentration Effect

- ***Proposition 2***: *Intuition*
- One customer of mass 1 yields either 0 or p as a payoff
- Two customers of mass ½ add the possibility of receiving the intermediate values of $p/2$. Less risk implies higher prices.

Pure Concentration Effect

- ***Missing Proposition***
  - Would like to show *p* and 1/*n* are inversely related for all values of *n*.
  - Numerical examples for Π^{α} and linear utility with a positive intercept yield this relationship.
  - Larger customer size makes a market more risky.
  - Would like to show increased risk reduces the price.

Market Size Effect

- Let each customer increase by the same proportion.
- ***Proposition 3.*** For a seller with constant relative risk aversion, proportionate increases in each customer has no effect on price.
  - $E(U) = (1-p)(pm)^{\alpha}$
  - $\partial E(U)/\partial p = (1-p)\alpha(pm)^{\alpha-1}m - (pm)^{\alpha}$
  - m is raised to the power α in each term.

Customer Mix Effect

- There are two different size customer.
- In some cases large customer gets smaller price than small customer.
- In other (unusual?) cases the larger customer gets the larger price.

Customer Mix Effect

– ***Proposition 4***. With a customer of mass m and a continuum of 1-m, the single customer gets the lower price.
  - Price to the continuum is ½
  - Price to the single customer < ½

Customer Mix Effect

- Numerical results using Π^{α} and two customers.
  - For $\alpha > z$, $z < ½$, large customer gets low price
  - For $\alpha < z$ prices reverse, large customer gets high price
  - For α very close to 0, large customer gets low price

  - z falls as the number of small customers increases
  - z falls as difference in customer size increases
  - Prices fall as α approaches 0

Customer Mix Effect

– Intuition for price results with linear utility

- Customers have mass $m_1 < m_2$, $m_1 + m_2 = 1$
- $U = a + b\Pi$ for $\Pi \geq \varepsilon$
- $= 0$ for $\Pi < \varepsilon$
- The case of α very close to zero is approximated by the case of $b = 0$ in the linear utility example
- For $b = 0$ the seller wants to maximize the probability of making sales at least as great as ε.
- $p_1^* = \varepsilon/m_1$, $p_2^* = \varepsilon/m_2$
- $p_1^* > p_2^*$ when $m_1 < m_2$

Customer Mix Effect

– Intuition for price results.

- The case of α around .2 or .3 is approximated by the case of *a* being large relative to *b* in the linear utility example.
- Heuristic: If the seller could "lock up" a sale to one customer, then she could essentially maximize expected profits from the other.
- Since the "locked up" customer yields ε, there is a larger opportunity cost to locking up the large customer
- $p_1^* = \varepsilon / m_1$, $p_2^* = \sim 1/2$

Customer Mix Effect

- Intuition for price results.
  - The case of α around .7 or .8 is approximated by the case of a being small relative to b in the linear utility example.
  - In this case there is little incentive to "lock up" a sale just to ensure obtaining a.
  - The pure size effect dominates and the large customer is offered a lower price than the small customer.

Reasons for Risk Averse Firms

- Owners are risk averse and do not have costless access to riskless capital markets.
- Owners hold equity to signal quality.
- Risk averse managers have compensation that depends on the firm’s profits.
- Firms will discontinue a project (and fire the manager) if it does not meet a hurdle rate.
- Sales people receive a bonus for reaching a specific sales level.

Waterbed Effects Check

- Constant Relative Risk Aversion
- Start with three identical firms and calculate pre-merger price.
- Let two firms merge and calculate new prices.
- Larger firm gets lower price. but both firms get lower price than pre merger prices.
- Merger makes entire market riskier resulting in lower prices for all.

Risk Neutrality and Costs

- Seller with upward sloping MC everywhere gives bigger customer higher price.
- Seller with downward sloping MC everywhere gives bigger customer lower price.
- Intuition: When firm has downward sloping MC, bigger customer generates good externality lowering all other costs so firm at the margin want to increase the probability that big customer buys, so lowers price relative to small customer.

Risk Neutrality and Costs

- With one customer
- $E(\Pi) = (1-p)(p - c(1))$
- $\partial E(\Pi)/\partial p = 1- 2p^* + c(1) = 0$
- With two customers (half the size) FOC at p^*
- $\partial E(\Pi)/\partial p_i = \{1- 2p^* + c(1)\} + [c(1) -2c(½)][1- 2p^*]$
  Curly bracket expression = 0, and $[1- 2p^*] < 0$

  If $[c(1) -2c(½)] > (<) 0$ small customers get low (high) price